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An investigation of the effects of wave propagations in an infinite membrane upon the motions of salt particles on the surface

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AN INVESTIGATION OF THE EFFECTS OF WAVE PROPAGATIONS IN AN INFINITE MEMBRANE UPON THE MOTIONS OF SALT PARTICLES ON THE SURFACE

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By

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APPROVED BY READING COMMITTEE COMPOSED OF THE FOLLOWING MEMBERS

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DATE: June, 1953.

PREFACE

The subject of this study in vibrational motion is derived from a broader problem originally conceived by Mr. Henry Esterly of the department of physics of the University of Louisville. His initial attention to this problem was aroused in conjunction with certain intensive investigations of elastic properties of, and vibrations in solid media. To him, therefore, is due sincere acknowledgment for his unreserved and continued guidance in initiating this project.

For invaluable aid in the analysis of mathematical procedure and the discussion of conclusions immense credit belongs to my director, Mrs. E. E. Mayo. Additional recognition should be granted Messrs. J. T. Hoffman, W. Bauer, C. Bryan, and M. F. Rabb, for technical assistance. I should like to express deepest appreciation for the willing counsel and criticism offered by the entire physics staff of the University of Louisville.

Louisville, Kentucky — Robert T. Maupin

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INTRODUCTION

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The large body of material already in print which describes the experimental observations of vibrations in membranes does not warrant the addition of mere descriptions to the classical theory of vibrations. One is, in fact, led immediately to assume agreement with the conclusion that the experimental disagreements with theory are due simply to overidealizations of the equations of motion. Some such overidealized notions are : The definition of the membrane as an infinitely thin nondissipative homogeneous lamina;[1] the assumption that the restoring force is due to a uniform and constant tension; and the disregard of the longitudinal component of tension (and hence of longitudinal displacement and velocity) as negligible.

Most experiments with vibrating membranes have been to achieve approximations of the above conditions for the purpose of securing measurable wave elements in terms of stationary waves established by reflection from the boundary. Yet none of the published reports seem to indicate anyone's having made the interesting "mistake" of producing an apparent standing wave pattern without reflection from the boundary. Thus, instead of striving for improved reflection to satisfy theoretical

1. See Rayleigh, J.W.S., The Theory of Sound, (London,1926) I, P. 306

predictions, we have in our research established a phenomena similar in appearance to the stationary wave[1] where absence of the reflected transverse wave is assured.

This absence of a reflected wave is observable to the naked eye and by microscopic inspection of the motions of the salt particles on the membrane. This effect is recorded on the photographic plates in the text exhibiting cases of apparent stationary wave pattern response limited to a central region of the membrane. In this effect is embodied the "infinite membrane" concept incorporated in the title. It is hoped that the suggested methods of approach to this problem might yield enlightment for the investigation of other similar vibrating systems.

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1. According to Rayleigh, "Vibrations are called stationary when the motion of each particle of the system is proportional to some function of time the same for all particles." The most general kind of stationary vibration may be regarded as due to the superposition of equal progressive vibrations whose directions of propagation are opposed. Conversely, two stationary vibrations may combine into a progressive one. See ibid., p.349

MATERIALS AND CONSTRUCTION OF APPARATUS

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The membrane system was constructed on a 3' x 4' (inside measurement) frame with nylon as the vibrating medium. The frame was of 2" x 4" oak wood, panelled around the top edge with slanted white pine over which the nylon membrane was stretched. The membrane was first stapled near the outside of the slant upper edge of the frame, and tension was then increased by tacking a wood strip along the inside edge of the slanted top of the frame. In stretching the membrane we applied tension manually with approximately radial symmetry. This differs from the usual method of stretching the membrane on a rectangular frame where the tension is conventionally applied on only two orthogonal directions. The nylon material itself revealed a slightly greater tensile strength in one direction, but otherwide satisfied the desired conditions of uniformity in weave and flexibility. This disparity did not affect the recorded values since all readings were taken along the same radial line. The average thread count was about 120 threads per inch (unstretched).

A stiff pin, replacing the diaphragm of a permanent magnet-type loudspeaker, afforded the sinusoidal driving force.

The variable audio-frequency oscillator was an R.C.A. Beat Frequency Oscillator, Type T M V - 134 - A, Serial 1168. Volts 110-120. Watts 15. Cycles 50-60.

The above variable oscillator was calibrated at each test frequency against a fixed frequency 1000 ~ Audio Oscillator Type 213, Serial No. 674. General Radio Co., U. of L. Laboratory No. E-1206. This standard frequency tuning fork was employed in the comparison method of calibration requiring the interpretation of Lissajous curves.

A Type 168 Oscillograph, Allen B. Dumont Labs, Inc., Serial No. 552 served as a reliable frequency comparison indicator.

A Knight Phonograph Amplifier was incorporated for the purpose of controlling the amplitude of the driving force to ensure no reflection of the impressed wave motion.

Common table salt proved to be the most effective powder for indicating the nodal patterns of the observed wave phenomena. Sand, powdered rock smalt, and sawdust were tried with less satisfactory results and discarded due to undesirable factors of particle size, weight and hardness.

A microscope (Gaertner 38 M/M E F L) was employed in viewing the motion of the salt particles on the membranes and in making thread counts.

The elastic properties of the nylon were investigated by means of a modified Young's Modulus apparatus. A strip of nylon was suspended in the place of the test wire of the Young's Modulus experiment. By a suitable system of hanging weights and recording the extension of the strip the ratio of tension to elongation was computed. The area density was determined by means of the laboratory analytical balance.

Other materials included are as follows:

1. An Astatic (Rochelle Salt Crystal) Phonograph Cartridge for detecting phase changes in longitudinal and transverse directions. (The cartridge was orientated for discriminate detection of the longitudinal wave motion. The needle was then bent through 90^{o} and the cartridge was properly rotated to detect motion in the transverse direction only.)
2. An additional amplifier, suitably designed to amplify the wave signals transmitted from the crystal cartridge placed on the membrane surface.

PRELIMINARY EXPERIMENTAL OBSERVATIONS

PRELIMINARY EXPERIMENTAL OBSERVATIONS

The following observations pertain to the preliminary performance of the designed apparatus as previously described.

The wave length of the observed wave motion is continuously inversely proportional to the frequency (See photographic plates).

The salt pattern symmetry is not a function of the boundary configuration. (This independence of nodal symmetry upon the boundary was aided by the manner of stretching, namely, with the end in view of attaining radial symmetry.) This method of applying tension is in contrast to that conventionally employed for rectangular frames, that is, in only two perpendicular directions parallel with the boundary faces.

Somewhat allied to the above observation is the noticeable dependence of the pattern distinctiveness upon the position in which the loudspeaker is orientated. The loudspeaker pin (the sinusoidal driving force) definitely had a greater lateral motion in one direction. This characteristic is to be expected from the definite though slight lever-like action of the pin support.

The application of an effective boundary, namely, by manual exertion, does not affect the pattern if applied outside the observable wave pattern. This observation was not made for a similar application of exertion within the pattern.

Even at nodal positions for the observed wave pattern a transverse component of particle (salt) motion is observed.

The salt particle distribution did not detectably alter the nodal pattern symmetry or the distances between the salt collections, but tended only to affect the radius of the area of response of the membrane to the driving force.

Repeated microscopic thread counts revealed very little variation of the elastic properties of the nylon with respect to humidity or temperature conditions. There is no doubt, however, that the prolonged stretching and changing of the tension by the impressed force affected the pattern reproducibility to some degree.

It might be stated, superficially, that the observed wave patterns conformed in grosser details to concentric ellipses, and in some cases very closely approximated circles, which would be predicted for the classical transverse vibrations of a nearly circular membrane. At lower frequencies and greater drive the membrane did exhibit familiar patterns due to reflection from the boundary.

METHOD OF APPROACH TO THE PROBLEM

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The first positive achievement of this project was the establishment of an apparent standing wave[1] propagation in an infinite medium. The maximum wave amplitude is easily controlled by means of a suitable amplifier. Damping reduces this amplitude satisfactorily with a decay in amplitude as a function of distance from the impressed driving source. This damping is well indicated by the observed motions of the salt particles. That the observed pattern is definitely not due to reflection is attested to by the continuously inverse relationship between pattern formation and frequency. It is common knowledge that for our effect to be due to reflection the distance from the source to the boundary would have to be equal to an integral multiple of one-half the wave length. If it is granted that we do have an apparent standing wave propagation in an infinite medium, we seek next a valid explanation of our observed phenomena.

The exhibition of the independence of the pattern formation upon particle distribution eliminates the suggestion that any such damping by the particles effects a "virtual boundary" on the membrane.

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1. By apparent standing wave we refer to a wave effect which is not visibly altered in space with respect to time. The herein proposed theory of the true nature of the effect will be clarified later.

Turning to an inspection of the particle motion on the membrane one grasps the prospective fruitfulness of describing the complete motions of different portions of the membrane in terms of the energy imparted to the particles at the surface. An attempt would then be advanced to quantize the vibrating system as satisfying the conditions governing the particle motions.

The expressions of the possible motions of the particles in the membrane are derived from a consideration of the stresses, strains and vibrations of a deformable body[1].

The equations of motion of an elastic body are given by[2],

$$\rho\frac{\partial^2\xi_i}{\partial t^2} = \sum_j \left[(\lambda+\mu)\frac{\partial^2\xi_j}{\partial x_i\,\partial x_j} + \mu\frac{\partial^2\xi_i}{\partial x_j^2}\right] = \text{Force}\left(\frac{1}{\text{volume}}\right)$$

where $dF_i = \sum_j \frac{\partial T_{ij}}{\partial x_j}\, dx_1\, dx_2\, dx_3$, ρ = density

$\frac{\partial^2\xi_i}{\partial t^2}$ = acceleration of volume element $dx_1\, dx_2\, dx_3$

$i = 1,2,3.$ μ = shear modulus of elasticity

given by T_{ij} = shearing stress $= \mu \tan\theta$

$$\theta = \tan^{-1}\left(\frac{\partial\xi_i}{\partial x_j}\right)$$

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1. For this treatment see Slater and Frank, Mechanics (New York, 1947), pp. 206-222. Also see Page, Introduction to Theoretical Physics (New York, 1935), pp. 173-179.
2. Slater and Frank, op cit., p. 218.

$\frac{\partial \xi_i}{\partial x_j}$ = pure strain (shearing)

λ = tension modulus of elasticity, is given by

$$\kappa = -\frac{1}{p}\frac{\Delta V}{V} = \frac{1}{\lambda + \frac{2}{3}\mu} = \text{compressibility.}$$

$$\frac{\Delta V}{V} = \text{dilatation} = \nabla \xi = \frac{\partial \xi_1}{\partial x_1} + \frac{\partial \xi_2}{\partial x_2} + \frac{\partial \xi_3}{\partial x_3}$$

This equation of motion for displacement employs Hooke's law which in tensor notation is expressed as follows:

$$T_{ij} = \lambda \delta_{ij} \sum_K \frac{\partial \xi_K}{\partial x_K} + \mu\left(\frac{\partial \xi_i}{\partial x_j} + \frac{\partial \xi_j}{\partial x_i}\right),$$

$\delta_{ij} = \sum_r \alpha_{ri}\alpha_{rj}$ for orthogonality - normalizing conditions

$= 1,\ i = j$

$= 0,\ i \neq j$

α_{ij} = direction cosines

The diagonal components of the above form of Hooke's law are,

$$T = (\lambda + 2\mu)\frac{\partial \xi_1}{\partial x_1} + \lambda\frac{\partial \xi_2}{\partial x_2} + \lambda\frac{\partial \xi_3}{\partial x_3}$$

$$0 = \lambda\frac{\partial \xi_1}{\partial x_1} + (\lambda + 2\mu)\frac{\partial \xi_2}{\partial x_2} + \lambda\frac{\partial \xi_3}{\partial x_3}$$

$$0 = \lambda \frac{\partial \xi_1}{\partial X_1} + \lambda \frac{\partial \xi_2}{\partial X_2} + (\lambda + 2\mu) \frac{\partial \xi_3}{\partial X_3},$$

$$\text{or, } T = Y \frac{\partial \xi_1}{\partial X_1} \quad \text{where } Y = (\lambda + 2\mu - 2\lambda\sigma).$$

$$\sigma = - \frac{\partial \xi_2 / \partial X_2}{\partial \xi_1 / \partial X_1} = \text{Poisson's ratio}$$

$$\approx \tfrac{1}{3} \text{ for most materials.}$$

$$Y = \text{Young's modulus.}$$

Our stated equations of motion may be simplified by considering a wave in which displacement is a function of distance in one direction only, let us say, of X, only. The components of our wave equation then become.

$$\rho \frac{\partial^2 \xi_1}{\partial t^2} = (\lambda + 2\mu) \frac{\partial^2 \xi_1}{\partial X_1^2}, \quad (\lambda + 2\mu) = Y \text{ if } \sigma \text{ is neglected.}$$

$= \text{longitudinal equation of motion.}$

$$\rho \frac{\partial^2 \xi_2}{\partial t^2} = \mu \frac{\partial^2 \xi_2}{\partial X_1^2} = \text{transverse equation of motion.}$$

$$\rho \frac{\partial \xi_3}{\partial t^2} = \mu \frac{\partial^2 \xi_3}{\partial X_1^2} = \text{transverse equation of motion.}$$

These equations exhibit two independent types of waves. The longitudinal wave can be propagated with a velocity $C_l = \sqrt{\frac{Y}{\rho}}$. The transverse waves are at right angles to the direction of propagation and the velocity is $C_t = \sqrt{\frac{\mu}{\rho}}$.

In our case we recognize the existance of two such waves in the membrane, longitudinal and transverse. These are expressed in the following forms:

$$\eta = A\cos\{\omega t - k_1 x\}\, e^{-\alpha x} = \text{transverse displacement},\ k_1 = \frac{2\pi}{\lambda_T}. \qquad (1.1)$$

$$\xi = B\cos\{\omega t - k_2 x\}\, e^{-\beta x} = \text{longitudinal displacement},\ k_2 = \frac{2\pi}{\lambda_L}. \qquad (1.2)$$

These are the two motions of the membrane and hence those which may be imparted to the salt particles.

At each point in the membrane the transverse velocity will at some time be zero. At each point in the membrane the longitudinal velocity will at some time be zero. At certain points in the membrane, there should exist a phase relationship between the longitudinal and transverse waves such that the longitudinal and transverse velocities will be zero simultaneously. And at sufficiently high frequencies the recurrence of such simultaneous zero wave Instantaneous velocities should be of sufficient periodicity to establish "pseudo-stationary" waves. Perhaps a better manner of expressing this phenomena is suggested by the observation that there may be vertical separation of the salt from the membrane even at the places where it collects. Therefore, the salt forms at the places where it leaves the screen with zero horizontal velocity. This last consideration is significant for considering the precise nature of the particle-membrane vibrating system.

Returning to the expressions for the displacements and velocities of the two waves in the membrane we state,

$$\dot{\eta} = -\omega A \sin\left\{\omega t - \frac{2\pi x}{\lambda_T}\right\} e^{-\alpha x} = \text{transverse particle velocity}, \tag{1.3}$$

$$= 0 \text{ for } \left\{\omega t - \frac{2\pi x}{\lambda_T}\right\} = n\pi, \; n = \text{integer}.$$

$$\dot{\xi} = -\omega B \sin\left\{\omega t - \frac{2\pi x}{\lambda_L}\right\} e^{-\beta x} = \text{longitudinal particle velocity}. \tag{1.4}$$

$$= 0 \text{ for } \left\{\omega t - \frac{2\pi x}{\lambda_L}\right\} = m\pi, \; m = \text{integer}.$$

To determine the spatial quantization for the conditions of observed particle motion, therefore, we eliminate ωt from the two above terms in the brackets, for consideration of the "space frequency" portion of the functions, namely, $\frac{2\pi x}{\lambda_T}$ and $\frac{2\pi x}{\lambda_L}$.

We obtain, $$\omega t = n\pi + \frac{2\pi x}{\lambda_T} = m\pi + \frac{2\pi x}{\lambda_L}. \tag{2.1}$$

$$2x\left(\frac{1}{\lambda_T} - \frac{1}{\lambda_L}\right) = m - n, \text{ or, } x = \frac{m-n}{2}\left(\frac{\lambda_L \lambda_T}{\lambda_L - \lambda_T}\right) \tag{2.2}$$

1. The terms $e^{-\alpha x}$, $e^{-\beta x}$ are introduced to satisfy the damped nature of the waves, and are the respective damping factors to be experimentally determined.

In terms of frequency, $X = \frac{p}{2}\left(\frac{C_L C_T}{C_L - C_T}\right) \cdot \text{frequency}^{-1}$ defines positions where the salt collects. $p = (m-n) = \text{integer}$

C_L = velocity of longitudinal wave propagation.
C_T = velocity of transverse wave propagation.

It would then be left to determine the values of C_l and C_t to check the validity of our spatial quantization of the particle-membrane system.

Another attempt to explain the particle distributions considered the kinetic energy of the membrane waves, which should be given by -

$$K.E. = \frac{m}{2}(\dot{\eta}^2 + \dot{\xi}^2) = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t - k_1 X) + \frac{1}{2} m\omega^2 B^2 \sin^2(\omega t - k_2 X). \quad (3.1)$$

$$\frac{\partial(K.E.)}{\partial X} = -m\omega^2 A^2 k_1 \sin(\omega t - k_1 X)\cos(\omega t - k_1 X)$$
$$- m\omega^2 B^2 k_2 \sin(\omega t - k_2 X)\cos(\omega t - k_2 X) = 0 \quad (3.2)$$

for minimum value if $\frac{\partial^2(K.E.)}{\partial X^2} > 0$.

$$\frac{\partial(K.E.)}{\partial X} = -\frac{m\omega^2 A^2}{2}\sin 2(\omega t - k_1 X) - \frac{m\omega^2 B^2}{2}\sin 2(\omega t - k_2 X) \quad (3.3)$$

$$= 0 = R\sin 2(\omega t - k_1 X) + \sin 2(\omega t - k_2 X).$$

$$R = \frac{A^2 k_1}{B^2 k_2}$$

The conditions satisfying the above minima are,

$$2\omega t - 2k_1 X = 0 \pm 2n\pi, \text{ and } 2\omega t - 2k_2 X = 0 \pm 2m\pi, \quad (3.4)$$

$$\text{or, } \omega t = k_1 X \pm n\pi = k_2 X \pm m\pi. \quad X = \frac{m-n}{2}\left(\frac{\lambda_L \lambda_T}{\lambda_L - \lambda_T}\right). \quad (3.5)$$

This result agrees with that of our first consideration.

Expanding equation (3.3), we obtain

$$\frac{\partial(K.E.)}{\partial x} = R \sin 2\omega t \cos 2k_1 x - R \cos 2\omega t \sin 2k_1 x$$
$$+ \sin 2\omega t \cos 2k_2 x - \cos 2\omega t \sin 2k_2 x = 0 \,. \quad (4.1)$$

Collecting coefficients of $\sin 2\omega t$ and $\cos 2\omega t$, we obtain

$$\sin 2\omega t(R \cos 2k_1 x + \cos 2k_2 x) - \cos 2\omega t(R \sin 2k_1 x + \sin 2k_2 x) = 0 \quad (4.2)$$

if

$$R \cos 2k_1 x + \cos 2k_2 x = \cos 2\omega t, \quad (4.3)$$

$$R \sin 2k_1 x + \sin 2k_2 x = \sin 2\omega t. \quad (4.4)$$

Squaring and adding we get $\cos 2x(k_1 - k_2) = -\frac{R}{2}$, (4.5)

$$x = \frac{\Phi \pm 2n\pi}{4\pi}\left(\frac{\lambda_L \lambda_T}{\lambda_L - \lambda_T}\right). \quad \Phi = \cos^{-1}\left(-\frac{R}{2}\right). \quad (4.6)$$

Other conditions satisfying equation (4.2) are,

$$R \cos 2k_1 x + \cos 2k_2 x = 0 \quad (5.1)$$

$$R \sin 2k_1 x + \sin 2k_2 x = 0 \quad (5.2)$$

Dividing these equations, we get

$$\cot 2k_1 x = \cot 2k_2 x \,, \text{ or,} \quad (5.3)$$

$$2k_1 x + n\pi = 2k_2 x + m\pi. \quad (5.4)$$

$$x = \frac{(m-n)\pi}{2(k_1 - k_2)} = \frac{p}{4}\left(\frac{\lambda_L \lambda_T}{\lambda_L - \lambda_T}\right). \quad (5.5)$$

This result predicts exactly twice as many positions for particle ring formations as predicted by the velocity considerations. This result also includes the same points predicted earlier.

EXPERIMENTAL DETERMINATIONS AND MEASUREMENTS

To Determine the Velocity of Longitudinal Wave Propagation in the Membrane in Terms of the Elastic Properties of the Material

The velocity of propagation of the longitudinal wave is given as $C_1 = \sqrt{\frac{Y}{\rho}}$, Y = Young's Modulus $= \left(\frac{\text{dynes}}{\text{cm}^2}\right)$. ρ = density $= \left(\frac{\text{gm}}{\text{cm}^3}\right)$. Young's Modulus is defined by Hooke's law as the ratio of the applied stress to the resulting extensional strain, or $Y = \left(\frac{\text{Applied Force}}{\text{Area}} \times \frac{\text{Unstrained length}}{\text{Elongation}}\right)$, both stress and strain assumed small enough to prevent permanent deformation. The table below records the extensions of elongations with the corresponding stresses.

Plate No. 1.

	Test 1.			Test 2		
Force Applied (gm.)	Position of Extension Adding Weights (mm)	Position of Extension Removing Weights (mm)	Average Position of Extension (mm)	Position of Extension Adding Weights (mm)	Position of Extension Removing Weights (mm)	Average Position of Extension (mm)
1050	12.65	13.16	12.905	13.16		
1250	13.15	14.25	13.70	13.66	14.69	14.175
1450	14.14	15.07	14.605	14.81	15.66	15.235
1650	15.04	15.77	15.405	15.51	16.06	15.815
1850	15.96	15.96	15.960	16.75	16.75	16.750

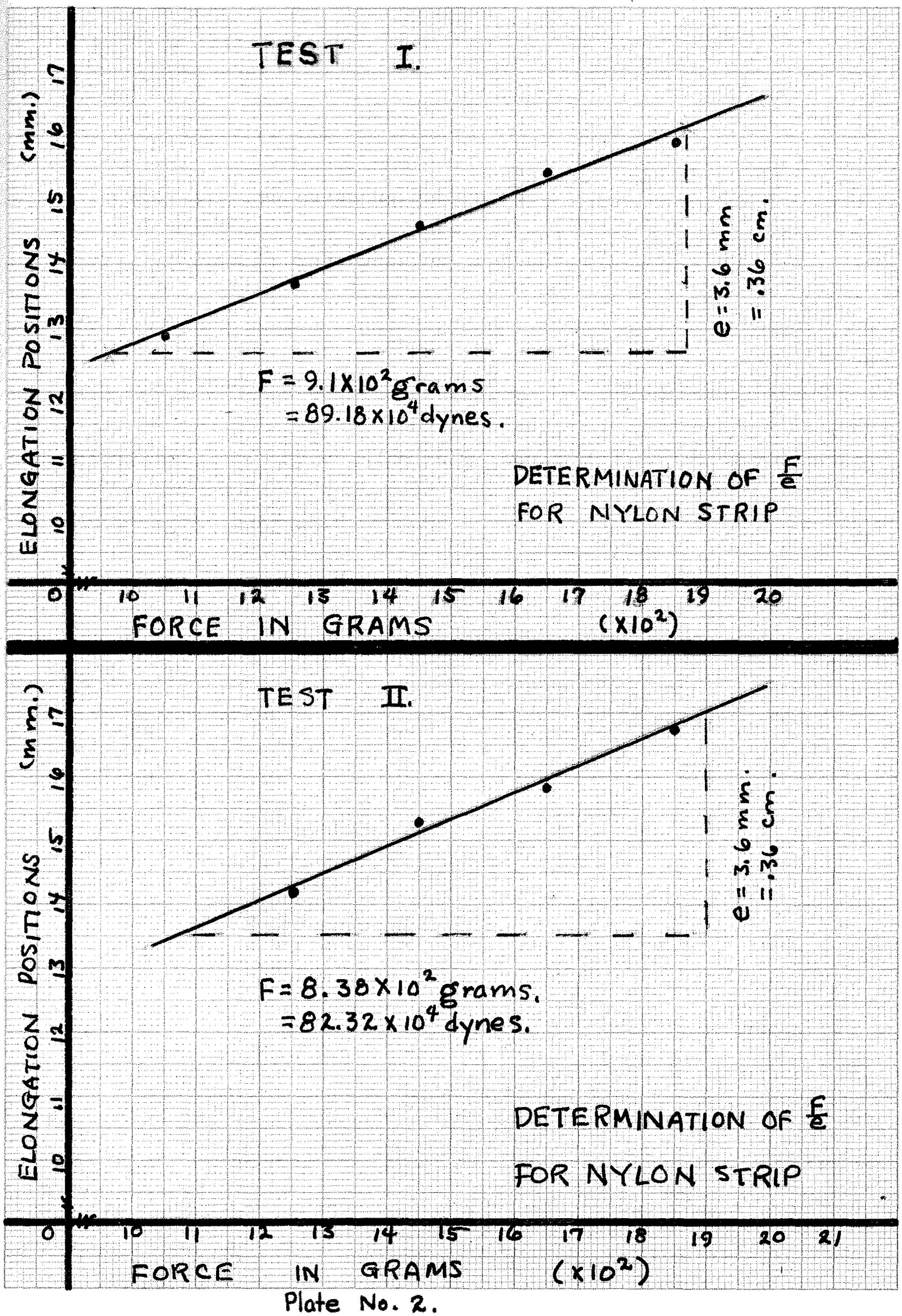

TEST I.
ELONGATION POSITIONS (mm.)
10 11 12 13 14 15 16 17
F = 9.1 X 10² grams
= 89.18 X 10⁴ dynes.
e = 3.6 mm
= .36 cm.
DETERMINATION OF F/e
FOR NYLON STRIP
0 10 11 12 13 14 15 16 17 18 19 20
FORCE IN GRAMS (X10²)
TEST II.
ELONGATION POSITIONS (mm.)
10 11 12 13 14 15 16 17
F = 8.38 X 10² grams.
= 82.32 X 10⁴ dynes.
e = 3.6 mm.
= .36 cm.
DETERMINATION OF F/e
FOR NYLON STRIP
0 10 11 12 13 14 15 16 17 18 19 20 21
FORCE IN GRAMS (X10²)

Plate No. 2.

Thus for two tests we obtain $\left(\frac{F}{e}\right)_1 = \frac{89.18 \times 10^4}{.360}$ dynes/cm $= 248.5 \times 10^4$ dynes/cm, and $\left(\frac{F}{e}\right)_2 = \frac{82.32 \times 10^4}{.360} = 228.5 \times 10^4$ dynes/cm. $\left(\frac{F}{e}\right)_{(Aver)} = 238.5 \times 10^4$ dynes/cm. Therefore $Y = \frac{F}{e}\frac{1}{a} = 238.5 \times \left(\frac{6.7}{\text{thickness}}\right) \times 10^4 = \frac{1595}{\text{thickness}} \left(10^4\right)$ dynes/cm^2 . $c_1 = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{1595/t \times 10^4}{36.38 \times 10^{-4}/t}} = 10^5 \sqrt{\frac{15.95}{36.38}}$ cm/sec $= 6.61 \times 10^4$ cm/sec.

Plate No. 3

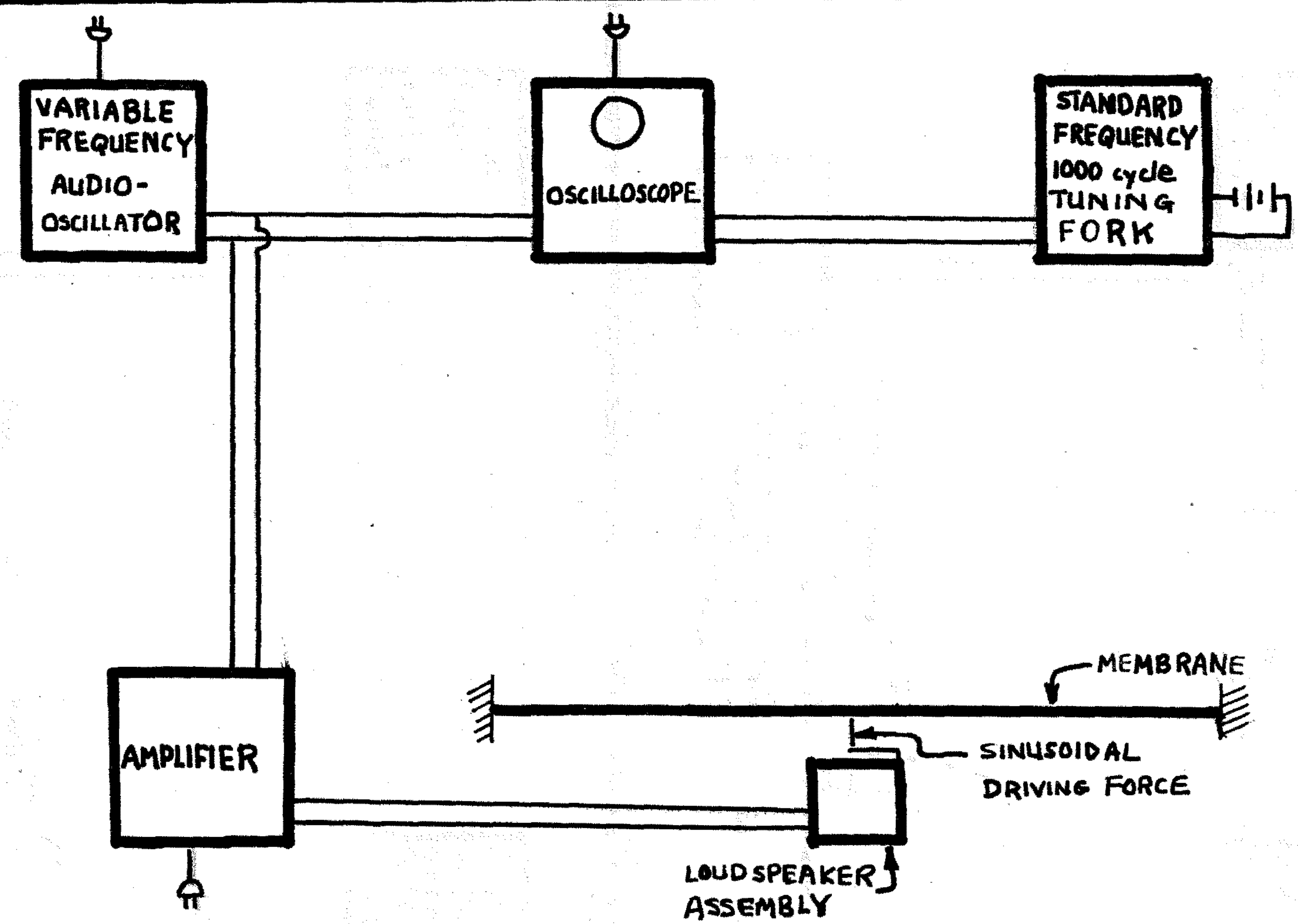
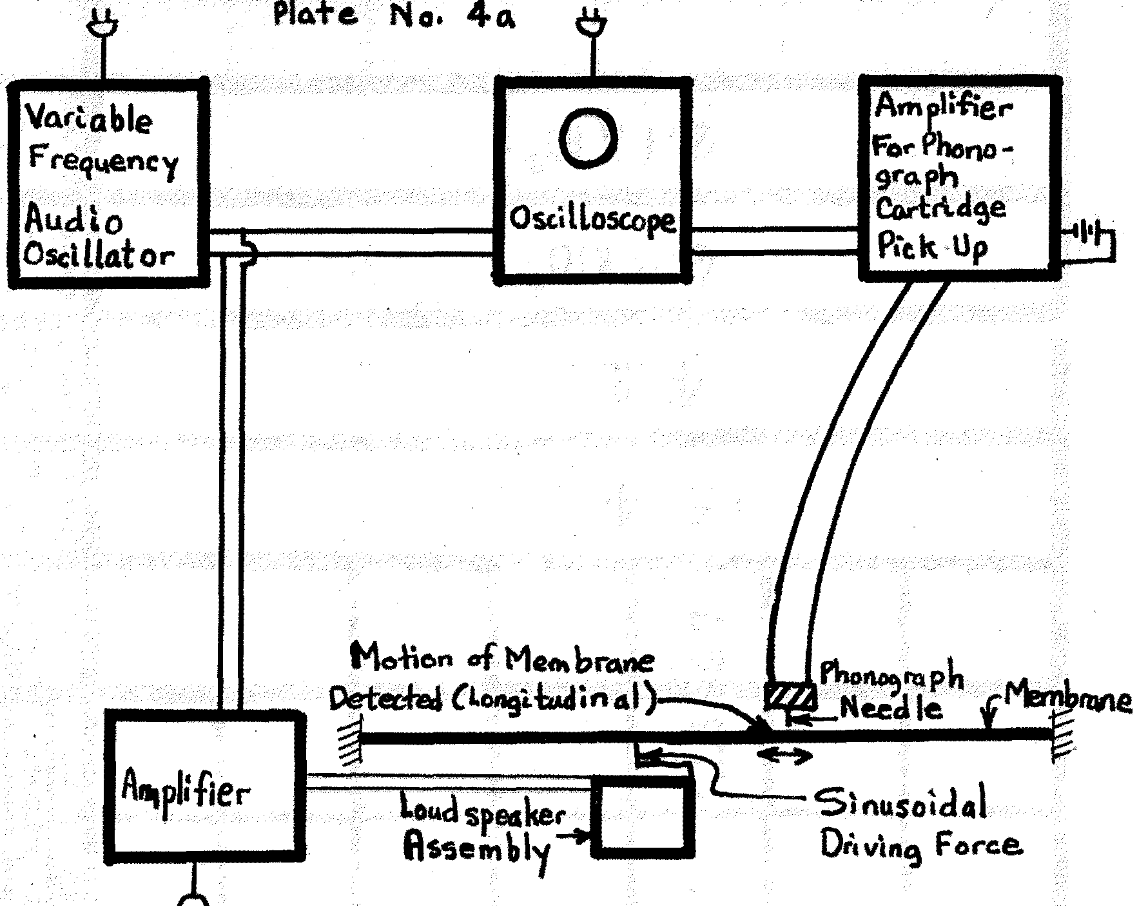


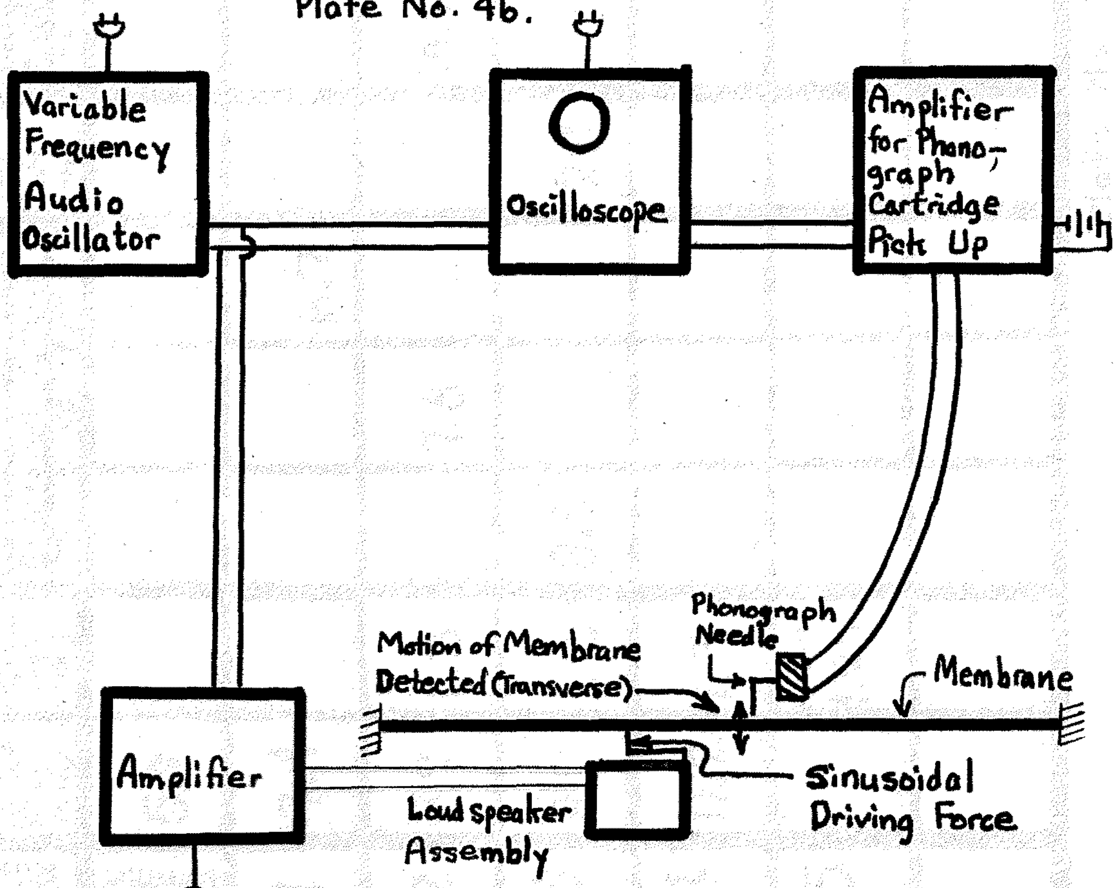
DIAGRAM OF MEMBRANE SYSTEM

Plate No. 4a



MEASUREMENT OF LONGITUDINAL WAVE LENGTHS

Plate No. 4b.



MEASUREMENT OF TRANSVERSE WAVE LENGTHS

Frequency = 5000 cycles/sec.

Observed Ring	Radial Position of Salt Rings (cm.)	Distance Between Adjacent Rings (cm.)	Position of Transverse Phase Repetitions — Oscilloscope Patterns			Distance Between 180° Phase shifts (cm.)	Position of Longitudinal Phase Repetitions — Oscilloscope Patterns			Distance Between 180° Phase shifts (cm.)	λ_L (cm.)	λ_T (cm.)	C_L cm/sec	C_T cm/sec	
			¬	O	⌐		¬	O	⌐						
Center	5.9														
1	7.4				6.7										
2	8.7	1.3		7.8			14.6								
3	9.9	1.2	8.8			2.1									
4	11.2	1.3							21.3	6.7	13.4	4.2	6.7×10^4	2.1×10^4	
5		Average 1.27													

Plate No. 5.

Frequency = 6000 cycles/sec.

Observed Ring	Radial Position of Salt Rings (cm)	Distance Between Adjacent Rings (cm)	Position of Transverse Phase Repetitions — Oscilloscope Patterns			Distance Between 180° Phase shifts (cm.)	Position of Longitudinal Phase Repetitions — Oscilloscope Patterns			Distance Between 180° Phase shifts (cm.)	λ_L (cm.)	λ_T (cm.)	C_L cm/sec	C_T cm/sec	
			⌒	O	⌐		⌒	O	⌐						
Center															
1	8.1			6.7											
2	9.2	1.1			7.6		16.2								
3	10.2	1.0		8.2		1.50									
4	11.2	1.0	9.6			2.0			21.6	5.4	10.8	3.66	6.48×10^4	2.29×10^4	
5	12.2	1.0		10.2		2.0									
		Average 1.025				Average 1.83									

Plate No. 6.

Frequency = 7000 cycles / sec.

Observed Ring	Radial Position of Salt Rings (cm.)	Distance Between Adjacent Rings (cm.)	Position of Transverse Phase Repetitions — Oscilloscope Patterns			Distance Between 180° Phase Shifts (cm.)	Position of Longitudinal Phase Repetitions — Oscilloscope Patterns			Distance Between 180° Phase Shifts (cm.)	λ_L (cm.)	λ_T (cm.)	C_L cm/sec	C_T cm/sec	
			¬	◡	⌐		¬	◡	⌐						
Center	5.9														
1				6.8											
2	8.0		7.5												
3	8.90	.90		8.4		1.6			13.6						
4	9.85	.95			9.4	1.9					10.2	3.32	7.14×10^4	2.32×10^4	
5	10.75	.90		9.9		1.5	18.7			5.1					
6	11.65	.90				Average									
		Average .91				1.66									

Plate No. 7

Frequency = 8000 cycles/sec.

Observed Ring	Radial Position of Salt Rings (cm.)	Distance Between Adjacent Rings (cm.)	Position of Transverse Phase Repetitions — Oscilloscope Pattern ⌝	◡	⌜	Distance Between 180° Phase shifts (cm.)	Position of Longitudinal Phase Repetitions — Oscilloscope Patterns ⌝	◡	⌜	Distance Between 180° Phase Shifts (cm.)	λ_L (cm.)	λ_T (cm.)	C_L cm/sec	C_T cm/sec	
center	6.0														
1	6.9		6.8												
2	7.7	.80		7.2			18.8								
3	8.5	.80			7.9	1.1									
4	9.3	.80		8.6		1.4			22.8	4.0	8.0	2.66	6.4×10^4	2.12×10^4	
5	10.1	.80	9.4			1.5									
		Average .80				Average 1.33									

Plate No. 8.

Plate No. 9.
Frequency = 2000 cycles/sec.

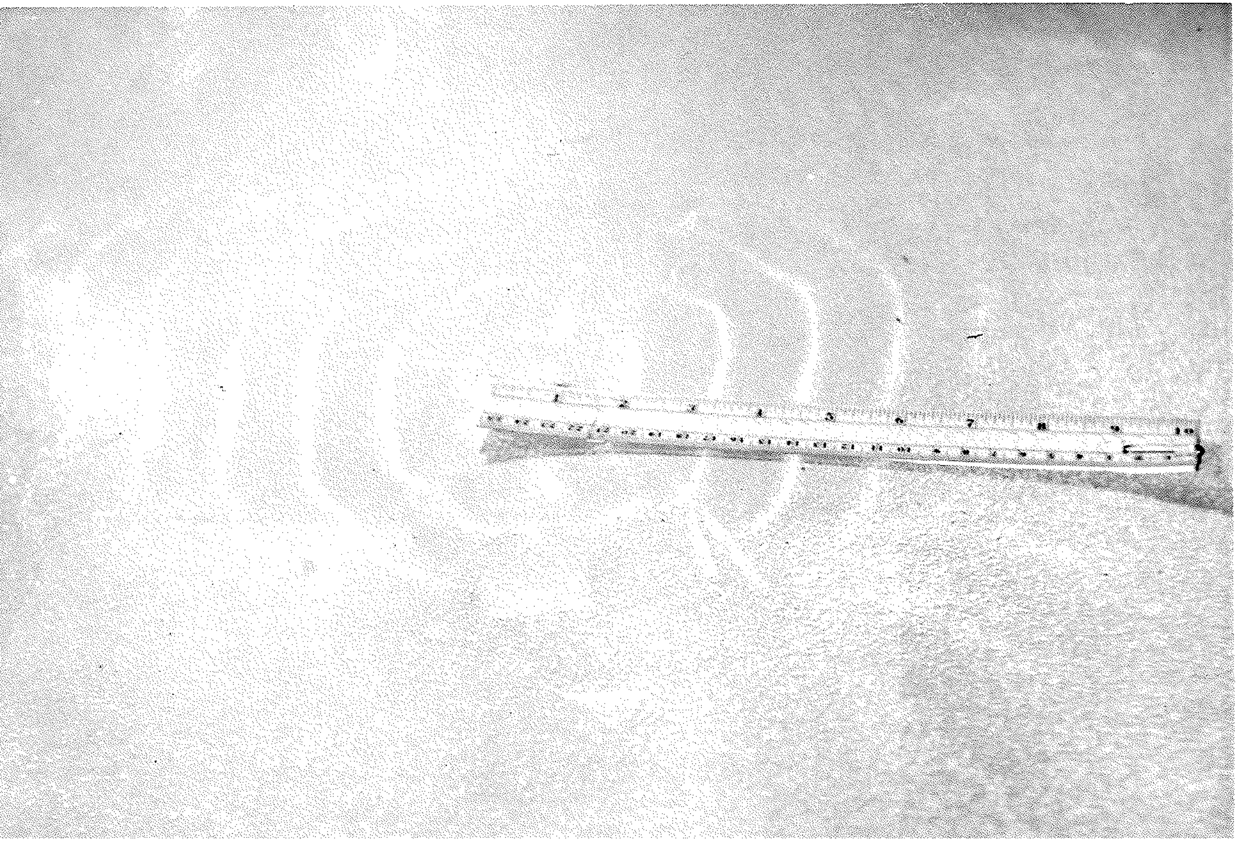


Plate No. 10.

Frequency = 3000 cycles/sec.

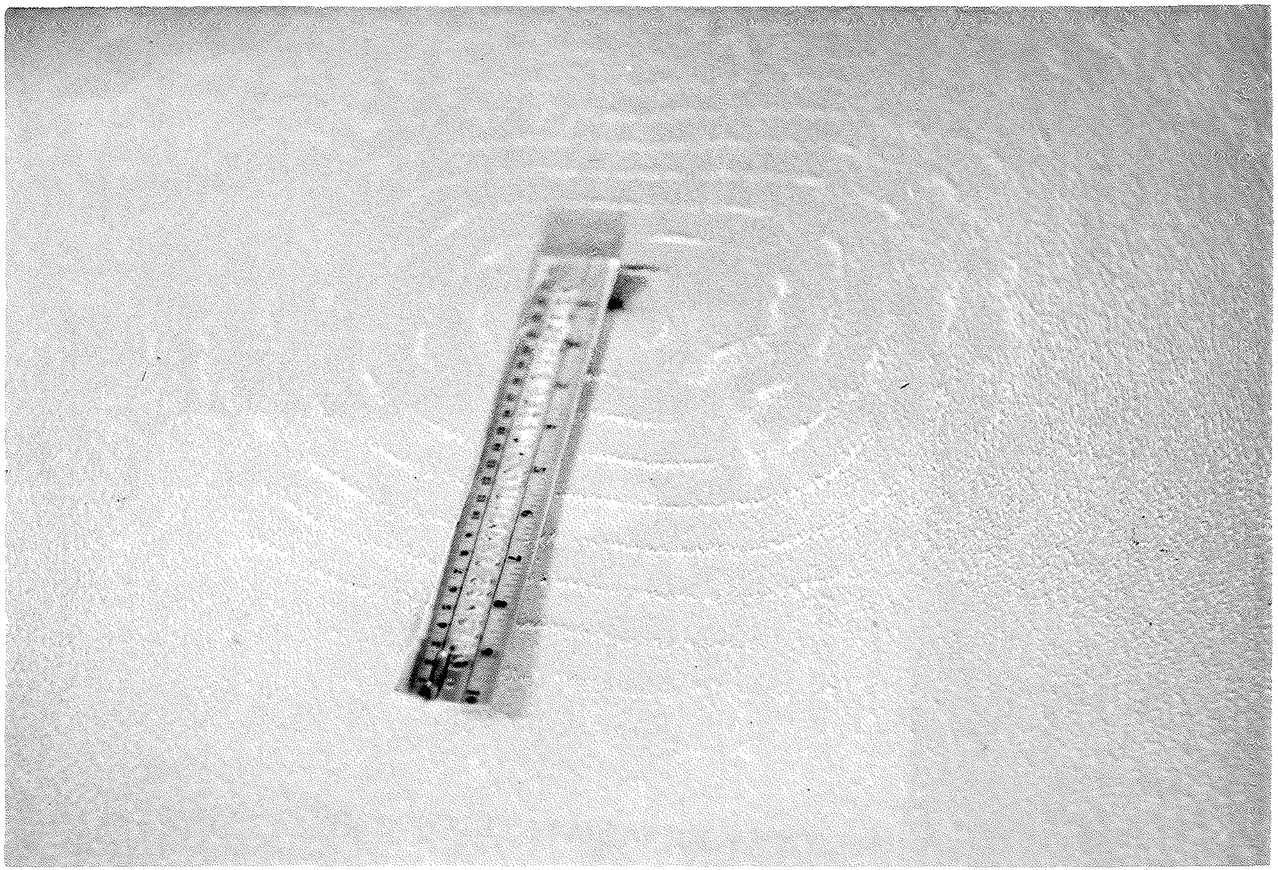


Plate No. 17.

Frequency = 4000 cycles/sec.

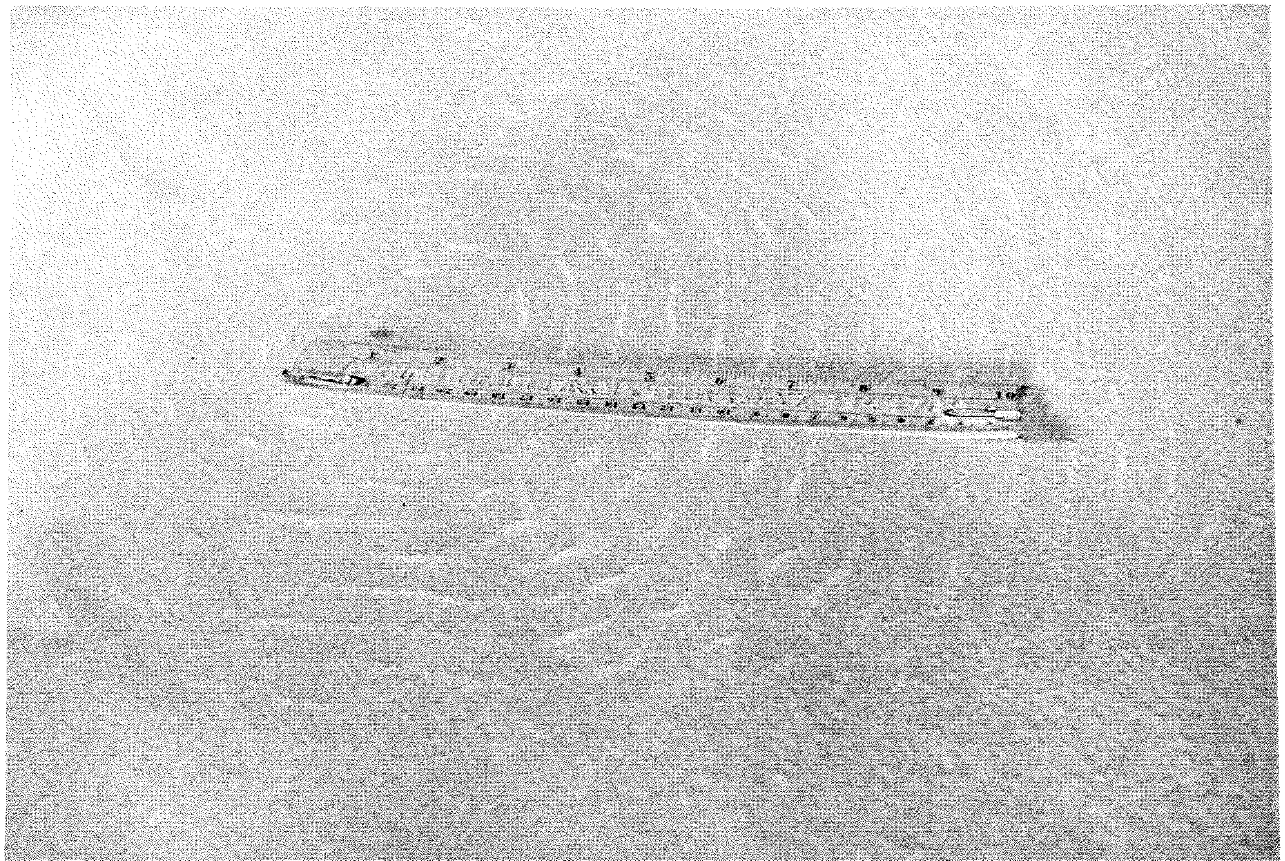


Plate No. 12.
Frequency = 5000 cycles / sec.

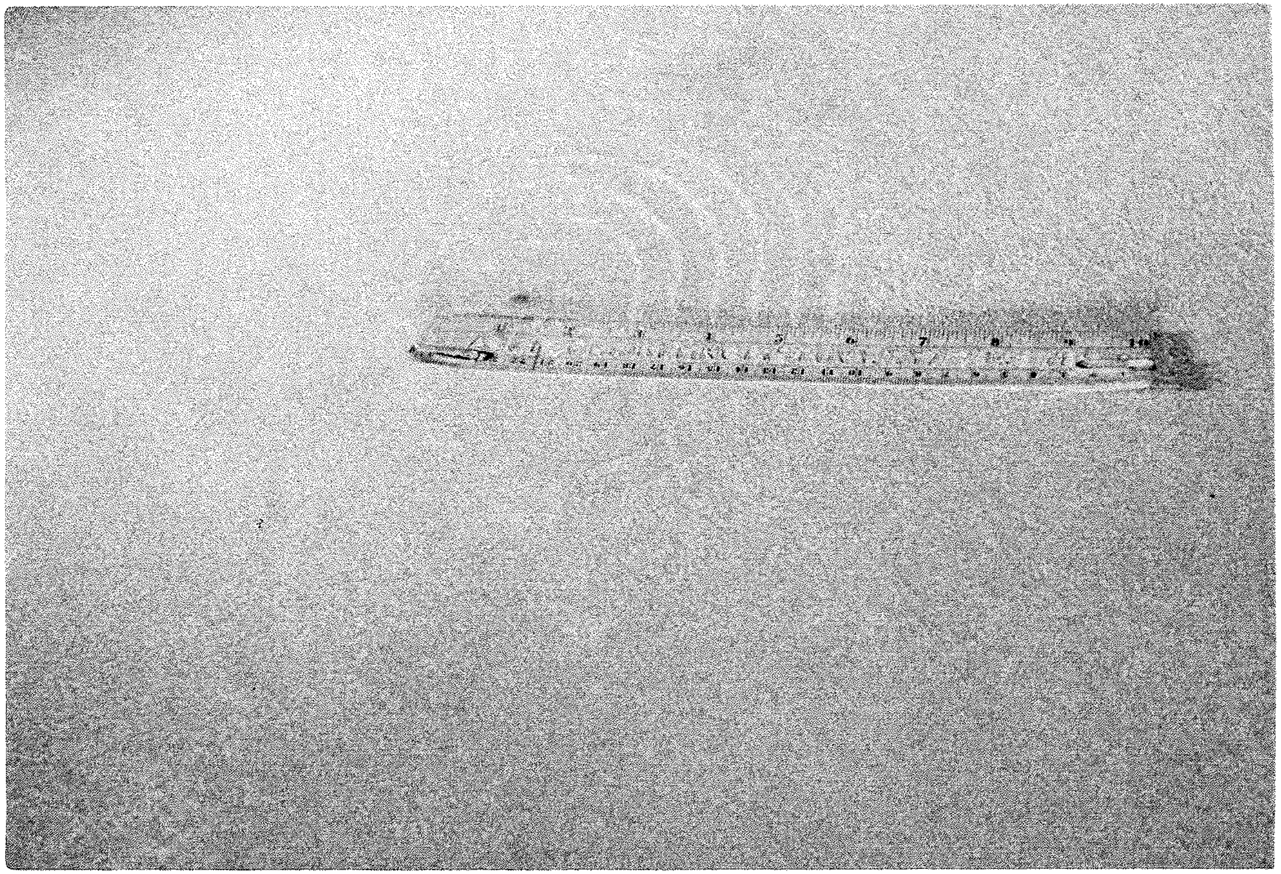


Plate No. 13.
Frequency = 6000 cycles/sec.

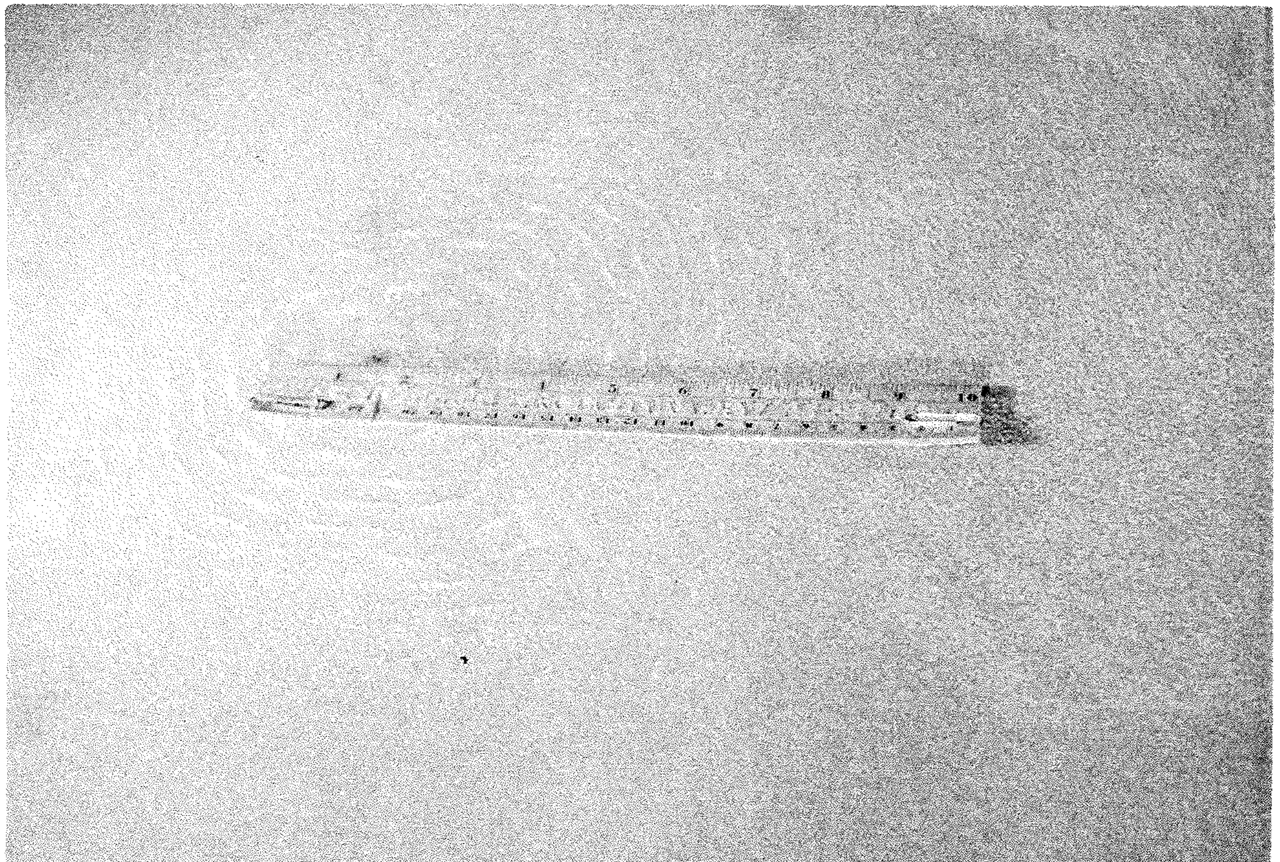


Plate No. 14
Frequency = 7000 cycles/sec.

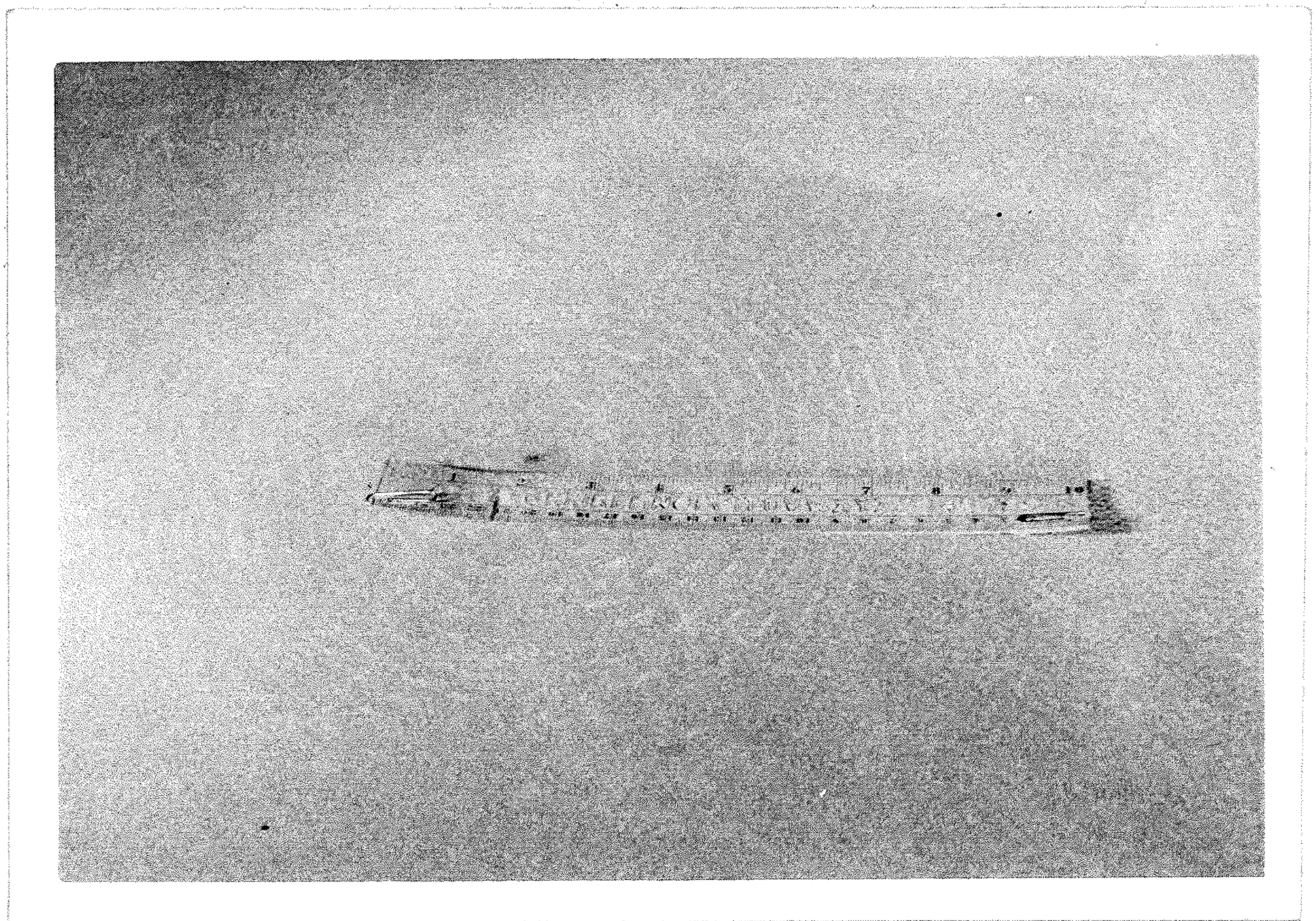
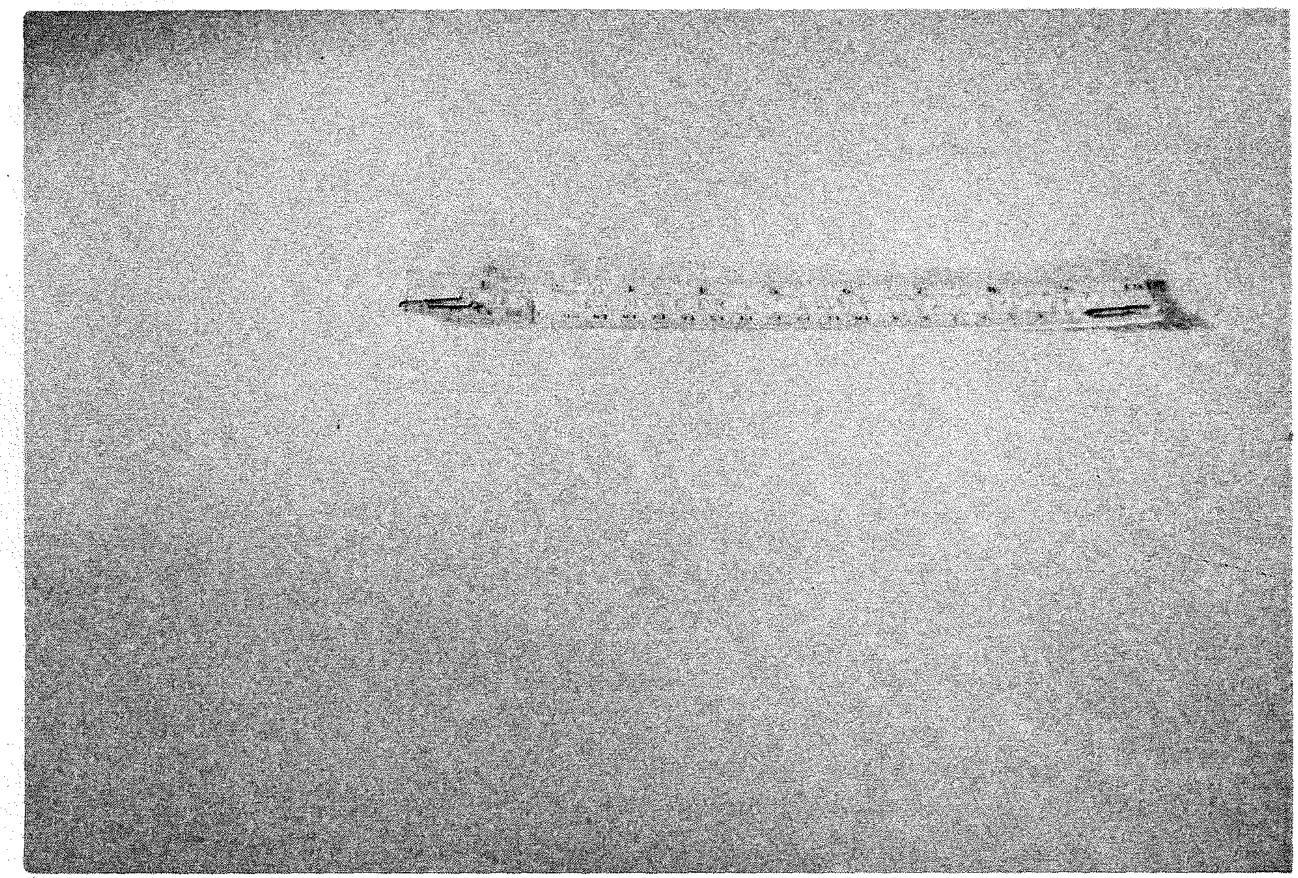


Plate No. 15
Frequency = 8000 cycles/sec.



CORRELATION OF

EXPERIMENTAL RESULTS

AND

THEORETICAL PREDICTIONS

INTERPRETATION OF WAVE LENGTH DATA

The respective values of C_L for frequencies from 5000 cycles/sec. to 8000 cycles/sec., inclusive, were: 6.70×10^4 cm/sec., 6.48×10^4 cm/sec., 7.14×10^4 cm/sec., 6.40×10^4 cm/sec., yielding an average $C_L = 6.73 \times 10^4$ cm/sec. This value of $C_L = \lambda_L \cdot$(frequency) compares with the value of C_L obtained by computing $C_L = \sqrt{\frac{Y}{\rho}} = 6.61 \times 10^4$ cm/sec. The two values average to 6.67×10^4 cm/sec.

The computed values of $C_T = \lambda_T \times$ (frequency) were: 2.1×10^4 cm/sec., 2.29×10^4 cm/sec., 2.32×10^4 cm/sec., 2.12×10^4 cm/sec. The average value of C_T is 2.20×10^4 cm/sec. We were unable to compare this value of C_T with the value $C_T = \sqrt{\frac{\text{Tension in dynes}}{\text{area density (cm)}}}$, obtained in terms of the elastic constants of the material.

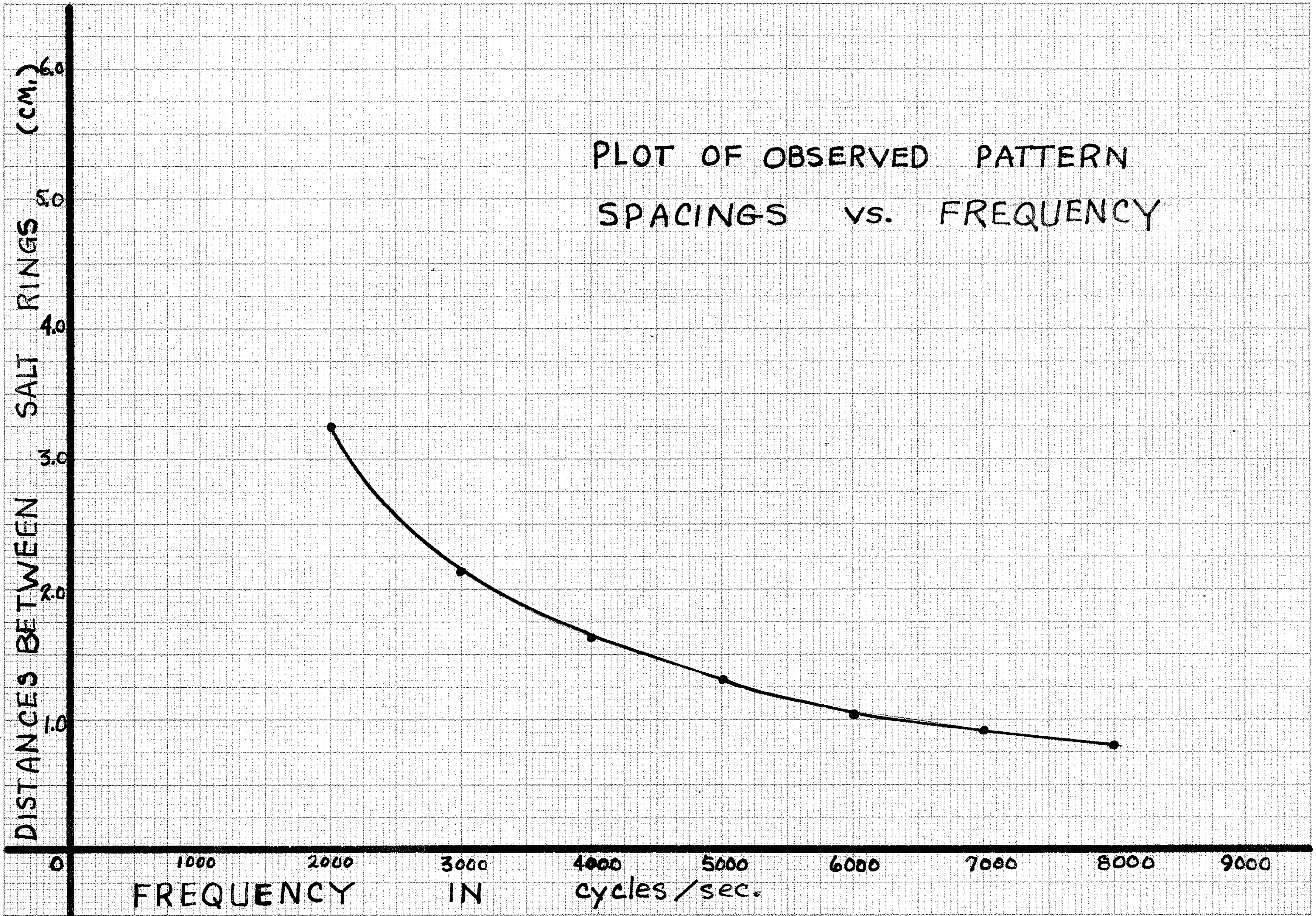

PLOT OF OBSERVED PATTERN
SPACINGS vs. FREQUENCY
DISTANCES BETWEEN SALT RINGS (CM.)
6.0
5.0
4.0
3.0
2.0
1.0
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
FREQUENCY IN cycles/sec.

PLATE NO. 16

COMPUTATIONS OF ALLOWED PATTERN RADII IN TERMS OF EXPERIMENTALLY DETERMINED VALUES OF AND

Using the relationship $X = \frac{P\,\lambda_L \lambda_T}{2(\lambda_L - \lambda_T)}$:

I(a) For $f = 5 \times 10^3$ cycles/sec., $\frac{\lambda_L \lambda_T}{\lambda_L - \lambda_T} = 6.117$ cm.

$X_{theoretical} = (.5, 1.0, 1.5, \ldots)\ 6.117$ cm.

$= 3.058, 6.117, \ldots$ cm. $= (2.39)\ X_{observed}$.

II(a) For $f = 6 \times 10^3$ cycles/sec., $\frac{\lambda_L \lambda_T}{\lambda_L - \lambda_T} = 5.53$ cm.

$X_{theo.} = 2.765, 5.53, \ldots$ cm. $= (2.6) X_{obs.}$.

III(a) For $f = 7 \times 10^3$ cycles/sec., $\frac{\lambda_L \lambda_T}{\lambda_L - \lambda_T} = 4.93$ cm.

$X_{theo.} = 2.47, 4.93, \ldots$ cm. $= (2.71)\ X_{obs.}$.

IV(a) For $f = 8 \times 10^3$ cycles/sec., $\frac{\lambda_L \lambda_T}{\lambda_L - \lambda_T} = 3.98$ cm.

$X_{theo.} = 1.99, 3.98, \ldots$ cm. $= (2.48) X_{obs.}$.

(Average)$X_{theo.} = (2.54) X_{obs.}$.

Using the relationship $X = \frac{P\,\lambda_L \lambda_T}{4\,(\lambda_L - \lambda_T)}$;

I(b) For $f = 5 \times 10^3$ cycles/sec.,

$X = (.25, 0.5, 1.0, \ldots)\ 6.117$ cm.

$= 1.529, 3.058, 4.59, 6.117, \ldots$ cm. $= 1.2\ X_{obs.}$.

II(b) For $f = 6 \times 10^3$ cycles/sec.,

$X_{theo.} = 1.382, 2.765, 4.14, 5.53 \ldots$ cm. $= 1.35\ X_{obs.}$.

III(b) For $f = 7 \times 10^3$ cycles/sec.,

$X_{theo.} = 1.23, 2.47, 3.7, 4.93, \ldots$ cm. $= 1.35\ X_{obs.}$.

IV(b) For $f = 8 \times 10^3$ cycles/sec.,

$X_{theo.} = .995, 1.99, 2.98, \ldots$ cm. $= 1.243\ X_{obs.}$.

(Average)$X_{theo.} = 1.285\ X_{obs.}$.

CONCLUSION

CONCLUSION

The form of the space equation, that we have developed to explain the behavior of the particles, expresses the desired continuity of dependence of radial particle collection distances upon the frequency. It was, therefore, disheartening to view the computed values for pattern spacings that averaged 2.54 and 1.285 times the observed spacings for velocity and kinetic energy considerations, respectively. One might like to attribute the discrepancies in our results to faulty measurements, since only a decrease in the values of λ's would be necessary to improve the theoretical values to the desired degree of accuracy. While inaccuracies in λ_r might be admitted, the agreement in the values of λ_ℓ as determined by the two methods described in the data section serves to justify the employment of the crystal phonograph cartridge method of segregated wave detection, where two independent types of wave motion co-exist simultaneously.

Though expressing the desired form of relationship between the variables involved, the resulting spacing equations imply to the investigator that there is definite need for refinement of the theoretical approach.

The consideration of the kinetic energy of the membrane was not carried further because a function of the type ($\sin^2 ax + R \sin^2 bx$), where R is a function of the frequency and amplitude ratios of the two existing waves, would require a more complete experimental investigation of the ratio values. It is suggested that the insight required for the complete description of this apparently discontinuous effect in a continuous medium should be further pursued both for purely theoretical and possibly practical contributions which might evolve.

BIBLIOGRAPHY

1. Slater, John C., and Nathaniel H. Frank, Mechanics. New York and London: McGraw-Hill Book Company, Inc., 1944.
2. Page, Leigh, Introduction to Theoretical Physics. New York: D. Van Nostrand Company, Inc., 1935.
3. Love, A. E. H., A Treatise on the Mathematical Theory of Elasticity. New York: Dover Publications, 1944.
4. Rayleigh, J. W. S., The Theory of Sound. London: MacMillan and Company, Limited, 1926.
5. Morse, Phillip M., VIBRATION AND SOUND. New York, Toronto, London: McGraw-Hill Book Company, Inc., 1948.
6. Kinsler, Lawrence E., and Austin R. Frey, Fundamentals of Acoustics. New York: John Wiley and Sons, Inc., London: Chapman and Hall, Limited, 1950.